

Issues in Gravitational Wave Data Analysis

Lee Samuel Finn

Physics and Astronomy, Northwestern University, Evanston IL 60208-3112, USA

Data analysis is the application of probability and statistics to draw inference from observation. Is a signal present or absent? Is the source an inspiraling binary system or a supernova? At what point in the sky is the radiation incident from? In these notes I want to address how two different statistical methodologies — Bayesian and Frequentist — approach the problem of statistical inference.

1 Introduction

There is a perception that Bayesian and Frequentist statistical methodologies are, at root, identical; that attempts to distinguish between them are sophistry; that, even if there are differences, they are only semantic and without any operational consequence.

These are all serious misconceptions. Bayesian and Frequentist statistical methodologies are inequivalent; they ask fundamentally different questions of the data in an attempt to draw inferences; and, because they ask different questions, the analyses undertaken are quantitatively and qualitatively different and lead to conclusions different in type and kind, even when regarding the same data set.

In §2 and §3 we pose the question “with what confidence can we conclude that, in the last hour, the gravitational waves from a new core collapse supernova in the Virgo cluster of galaxies passed through our gravitational wave detector?” In §2 we consider the question from a Bayesian perspective, while in §3 we consider it from a Frequentist perspective. We will find that this apparently straightforward question takes on a different meaning to the Frequentist and the Bayesian, leading each to respond in different ways.

Finally, in §4 we examine an example of a multi-detector data analysis problem — the detection of a stochastic gravitational wave signal in two detectors. A Frequentist analysis of this problem has been developed^{1,2,3,4} and is summarized first.^a In the following subsection we explore a Bayesian analysis of the same problem⁵, which leads to an analysis that is entirely unlike the Frequentist one. Together, the discussion in §§2–4 demonstrates the very real, operational differences between the Bayesian and Frequentist approaches to data analysis.

^aSee also Allen’s contribution to this proceedings.

2 Learning From Observation: Bayesian Data Analysis

With what confidence can we conclude that, in the last hour, the gravitational waves from a new core collapse supernova in the Virgo cluster of galaxies passed through our gravitational wave detector?

We don't approach this, or any other question, without some prior expectations. In this case, before examining the observations, our prior understanding of astrophysics leads us to expect, on average, one such core collapse every 4 months; consequently, we believe the probability is approximately 3.4×10^{-4} that in any given hour — including the last — gravitational waves from a new Virgo cluster supernova were incident on our detector.

Probability, as we have used it here, means *degree of belief*. In this instance, our degree of belief coincided with the *expected frequency* of supernova events; however, this need not be the case: we can assess degree of belief even when we can't assess relative frequency. For example, suppose that I have a coin that is known to be heavily biased toward either heads or tails. What is your degree of belief that, when I next flip the coin, it will land heads-up? Without telling you the direction of the bias, you can't evaluate the expected relative frequency of heads or tails. You can, however, quantify your degree of belief: having no more reason to believe that the bias is toward heads than towards tails, you have no more reason to believe that the coin will, when next flipped, land heads-up than that it will land heads-down. Your degree of belief in either alternative, then, is $1/2$.

One does not have to search either long or hard to find examples from astrophysics where probability as “degree of belief” exists and probability as “expected frequency” does not. For example, what is the probability that there exists a cosmological stochastic gravitational wave signal with a given amplitude and spectrum? In this case, “expected frequency” has no meaning: there is only one Universe, and it either does or does not have a stochastic gravitational wave background of given spectrum and amplitude.

After we examine the output of our gravitational-wave detector, our degree of belief in the supernova proposition may change: we may, on the basis of the observations, become more or less certain that radiation from a supernova passed through our detector. How do observations change our degree of belief in the different alternatives?

To explore how our degree of belief evolves with the examination of observations we need to introduce some notation:

$$H_0 = \left(\begin{array}{l} \text{proposition that gravitational waves from a} \\ \text{new supernova in the Virgo cluster } \textit{did not} \\ \text{pass through our detector in the last hour} \end{array} \right), \quad (1)$$

$$\mathcal{I} = \left(\begin{array}{l} \text{our prior knowledge of astrophysics, including} \\ \text{our best assessment of the supernova rate} \end{array} \right), \quad (2)$$

$$g = \left(\text{observations from our gravitational wave detector} \right), \quad (3)$$

$$P(A|B) = \left(\text{degree of belief in } A \text{ assuming that } B \text{ is true} \right), \quad (4)$$

$$\neg A = \left(\text{logical negation of proposition } A \right). \quad (5)$$

In this notation, $P(H_0|\mathcal{I})$ is the degree of belief we ascribe to the proposition that no gravitational waves from a core collapse supernova in the Virgo cluster passed through our detector in the last hour, given only our prior understanding of astrophysics; similarly, $P(H_0|g, \mathcal{I})$ is the degree of belief we ascribe to the same proposition, given *both* the observations g and our prior understanding of astrophysics.

To understand how $P(H_0|\mathcal{I})$ and $P(H_0|g, \mathcal{I})$ are related to each other we need to recall two properties of probability. The first is unitarity: probability summed over all alternatives is equal to one. In our example, the two alternatives are, given the observation g , a supernova occurred or it did not:

$$P(H_0|g, \mathcal{I}) + P(\neg H_0|g, \mathcal{I}) = 1. \quad (6)$$

The second property we need to recall is Bayes Law, which describes how conditional probabilities “factor”:

$$P(A|B, C)P(B|C) = P(A, B|C) = P(B|A, C)P(A|C). \quad (7)$$

Combining unitarity and Bayes Law it is straightforward to show that

$$P(\neg H_0|g, \mathcal{I}) = \frac{\Lambda(g)}{\Lambda(g) + P(H_0|\mathcal{I})/P(\neg H_0|\mathcal{I})} \quad (8)$$

where

$$\Lambda(g) = P(g|\neg H_0, \mathcal{I})/P(g|H_0, \mathcal{I}) \quad (9)$$

$$P(g|H_0, \mathcal{I}) = \left(\begin{array}{l} \text{probability that } g \text{ is a sample of} \\ \text{detector output when } H_0 \text{ is true} \end{array} \right) \quad (10)$$

$$P(g|\neg H_0, \mathcal{I}) = \left(\begin{array}{l} \text{probability that } g \text{ is a sample of} \\ \text{detector output when } H_0 \text{ is false} \end{array} \right) \quad (11)$$

The two probabilities $P(g|H_0, \mathcal{I})$ and $P(g|\neg H_0, \mathcal{I})$ depend on the statistical properties of the detector noise and the detector response to the gravitational wave signal. In some cases they can be calculated analytically; in other circumstances it may be necessary to evaluate them using, *e.g.*, Monte

Carlo numerical methods. Regardless of how one approaches data analysis — as a Bayesian or as a Frequentist — the detector must be sufficiently well characterized that these or equivalent quantities are calculable.

Equation 8 describes how our degree of belief in the proposition $\neg H_0$ evolves as we review the observations. If Λ is large compared to the ratio $P(H_0|\mathcal{I})/P(\neg H_0|\mathcal{I})$ then our confidence in $\neg H_0$ increases; alternatively, if it is small, then our confidence in $\neg H_0$ decreases. If Λ is equal to unity — *i.e.*, the observation g is equally likely given H_0 or $\neg H_0$ — then the posterior probability $P(H_0|g, \mathcal{I})$ is equal to the prior probability $P(H_0|\mathcal{I})$ and our degree of belief in H_0 is unchanged: we learn nothing from the observation.

More complex hypotheses are analyzed in the same way: if

$$H_0 = (\text{no signal was incident on the detector during the last hour})(12)$$

$$H_{\theta} = \left(\begin{array}{l} \text{the signal described by } \theta \text{ was incident} \\ \text{on the detector during the last hour} \end{array} \right), \quad (13)$$

where θ is some non-zero set of parameters, exhaust all possible alternative states of nature, then

$$P(\theta|g, \mathcal{I}) = \frac{\Lambda(g|\theta)}{\Lambda + P(H_0|\mathcal{I})/P(\neg H_0|\mathcal{I})} P(\theta|\neg H_0\mathcal{I}) \quad (14)$$

where

$$\Lambda(g|\theta) = P(g|H_{\theta}, \mathcal{I})/P(g|H_0, \mathcal{I}) \quad (15)$$

$$\Lambda(g) = \int d^n\theta \Lambda(g|H_{\theta}, \mathcal{I}) P(\theta|\neg H_0, \mathcal{I}) \quad (16)$$

$$P(\neg H_0|\mathcal{I}) = 1 - P(H_0|\mathcal{I}). \quad (17)$$

The alternative hypotheses represented by θ may represent the radiation from different sources (*e.g.*, supernovae, inspiraling compact binaries, stochastic signal, *etc.*), different numbers of sources (radiation from more than one example of a source, or from more than one kind of source), or details about the particular sources (signal amplitude, source sky position, inspiraling binary chirp mass, *etc.*).

We can now answer the question that began this section. As Bayesians, we understand confidence to mean *degree of belief* in the proposition that radiation originating from a new supernova in the Virgo cluster was incident on a particular detector during a particular hour. In response we calculate a quantitative assessment of our degree of belief in that proposition — the probability that the proposition is true.

3 Guessing Nature's State: Frequentist Data Analysis

With what confidence can we conclude that, in the last hour, the gravitational waves from a new core collapse supernova in the Virgo cluster of galaxies passed through our gravitational wave detector?

As before, we have the hypothesis H_0 and its logical negation, $\neg H_0$. The gravitational waves from a new Virgo cluster supernova either passed through our detector, or they did not. Our goal is to determine, as best we can, which of these two alternatives correctly describes what happened.

We decide which alternative is correct by consulting our observation g . Operationally, we adopt a rule or a procedure that, when applied to g , leads us to accept or reject H_0 . The question that began this section asks us to determine the most reliable rule or procedure.

There are many procedures that we can choose from. Some are silly: for example, always rejecting H_0 is a procedure. Similarly, accepting H_0 if a coin flip comes-up heads is a procedure. Some procedures are more sensible: we can calculate a characteristic amplitude from the observation (*e.g.*, a signal-to-noise ratio) and reject H_0 if the amplitude exceeds a threshold. Nature doesn't speak clearly and, often, some crucial information is hidden from us; so, no procedure will, in the end, be perfect and every rule will, on unpredictable occasions, lead us to erroneous conclusions. Still, some procedures are clearly better than others: the question is, how do we distinguish between them quantitatively?

Better procedures are those that are less likely to be in error; consequently, we focus on the error rate of different procedures. For our simple problem, where we want to decide only if we have or have not observed the radiation from a supernova (reject or accept H_0), there are two kinds of errors a decision procedure can make:

1. If no radiation is present (H_0 true), the rule may incorrectly lead us to conclude that radiation is present: a *false alarm*, or type I, error.
2. If radiation is present (H_0 false), the rule may incorrectly lead us to conclude that radiation is absent: a *false dismissal*, or type II, error.

The false alarm rate is generally denoted α while the false dismissal rate is denoted β .^b

The false alarm and false dismissal rates are *frequency probabilities*. If we have an ensemble of identical detectors simultaneously observing the same

^bWhen we have a more complex set of hypotheses — for example, when we are asked to choose between the set of alternative hypotheses H_0 and H_{θ} (cf. eq. 13) — there are additional measures of error: *e.g.*, the difference between the mean of the estimate of θ and its actual value, or the variance of the estimates, *etc.*

system for which H_0 (or $\neg H_0$) is true, and we apply our rule to each observation, then the limiting error rate is given by the false alarm (or dismissal) rate. When, in the last section, we wore our Bayesian hat, we understood probability to mean *degree of belief*; now, however, we understand probability to mean *limiting frequency of repeatable events*. The distinction in meaning is associated with the distinctly different interpretations of the question posed at the beginning of this and the last section.

We have seen that, even in the simple case at hand (a single hypothesis that we must accept or reject), there are at least two distinct kinds of errors that an inference procedure can make. Our measure of a rule's reliability thus involves at least two dimensions, and may involve more. How, then, do we order the rules to settle upon a best, or optimal, rule?

To rank rules we must reduce the several error measures that describe a procedure's performance to a single figure of merit. How we choose to do this depends on the nature of our problem. In our case, rules that distinguish between H_0 and $\neg H_0$ are characterized by their false alarm and false dismissal rates; consequently, our criteria for ranking rules should depend on relative intolerance to false alarms and false dismissals. For example, if we are testing for the presence of antibodies in an effort to diagnose and treat a serious illness, we might be very concerned to keep the false dismissal rate low, and not nearly as worried about a high false alarm rate: after all, a false dismissal might result in death, while a false alarm only in an unnecessary treatment with less serious repercussions. Judges or juries in criminal trials faces different concerns: false dismissals let criminals go free, while false alarms send the innocent to prison — neither alternative being very palatable. Finally, in the case of gravitational wave detection, we may (at least initially) be very concerned to avoid false alarms, even at the risk of falsely dismissing many real signals.

Thus, in order to provide a relative ranking of different inference procedures for detection or parameter estimation we must construct an *ad hoc* figure of merit reflecting the particular nature of the decision to be made. We term the best rule, under that *ad hoc* criteria, the “optimal” rule. “Optimality”, however, is a relative concept: if the criteria change, the “optimal” rule changes also. In the three examples given above, the criteria might be

- *medical diagnosis*: fix a maximum acceptable false dismissal rate and choose the rule that, among all rules whose false dismissal rate is so constrained, has the minimum false alarm rate;
- *criminal justice*: choose a rule whose weighted total error $\alpha \cos \phi + \beta \sin \phi$ is minimized (ϕ being a matter of personal choice for an individual judge or juror);

- *gravitational wave detection*: fix a maximum acceptable false alarm rate and choose the rule that, among all rules whose false alarm rate is so constrained, has the minimum false dismissal rate.

False alarm and dismissal rates describe our confidence in the long-run behavior of the associated decision rule. To understand the implications of this measure of confidence, suppose that we have not one, but N independent and identical detectors all observing during the same hour. We use the same test, with false alarm rate α and false dismissal rate β , on the observations made at each detector, and find that, of these N observations, m lead us (through our inference rule) to reject H_0 and $N - m$ lead us to accept H_0 . For a concrete example, suppose α is 1%, N is ten and m is three.

The probability of obtaining this outcome when the signal is absent (H_0 is true) is the probability of obtaining m false alarms in N trials, or

$$P(m|H_0, N) = \frac{N!}{(N - m)!m!} \alpha^m (1 - \alpha)^{N - m}. \quad (18)$$

In our example, $P(m|H_0, N)$ evaluates to 1.1×10^{-4} . It is thus very unlikely that we would have made this observation if the signal were absent. Does this mean we should conclude the signal is present with, say, 99.99% confidence?

No. $P(m|H_0, N)$ describes the probability of observing m false alarms out of N observations. When the signal is *present*, however (*i.e.*, when H_0 is false), there are *no* false alarms and both α and $P(m|H_0, N)$ are irrelevant. There are, however, $N - m$ false dismissals; thus, the relevant quantity is $P(m|\neg H_0, N)$, the probability of observing $N - m$ false *dismissals*:

$$P(m|\neg H_0, N) = \frac{N!}{(N - m)!m!} (1 - \beta)^m \beta^{N - m}. \quad (19)$$

If, in our example, the false dismissal rate β is 10%, then the probability of observing seven false dismissals out of ten trials is 8.7×10^{-5} .

The particular outcome of our example — three positive results out of ten trials — may be, overall, unlikely; however, it is more unlikely to have occurred when the signal is present than when it is absent. Despite the apparently overwhelming improbability of three false alarms in ten trials, it is nevertheless, slightly more likely than the alternative of seven false dismissals in ten trials.

We can now answer the question that began this section. As Frequentists, we understand the question to ask for the overall error rates of the best general procedure for deciding between the alternative hypotheses. We thus calculate the error rates for different inference rules, choose appropriate criteria for ranking the different rules, and find the best rule and its corresponding error rates.

Contrast this with our understanding of the identically worded question posed to us as Bayesians. As Bayesians, we understood confidence to mean the degree of belief that we should ascribe to alternative hypotheses; as Frequentists, we understand confidence to refer to the overall reliability of our inference procedure. As Bayesians we responded with a quantitative assessment of our degree of belief in the alternative hypotheses, *given a particular observation made in a particular detector over a particular period of time*; as Frequentists, we responded with an assessment of the relative frequency with which our rule errs given each alternative hypothesis. As Bayesians, we did not make a choice between alternative hypotheses; rather, we rated them as more or less likely to be true in the face of a particular observation. As Frequentists, on the other hand, we do make choices and our concern is with the error rate of our procedure for choosing, averaged over many different observations and many different decisions.

Frequentist analyses have particular utility when it is possible to make repeated observations on identical systems: *e.g.*, particle collisions in an accelerator, where each interaction of particle bunches is an “experiment.” Bayesian analyses, on the other hand, are particularly appropriate when the observations or experiments are non-repeatable: *e.g.*, when the sources are, like supernovae, non-identical and destroy themselves in the process of creating the signal. In the latter case, we are generally particularly interested in the properties of the individual systems and would prefer a measure of the relative degree of belief that we should ascribe to, for example, the proposition that the signal originated from different points in the sky.

4 Example: Data Analysis for Stochastic Signals

As an example of how Bayesian and Frequentist statistical methodologies lead to quantitatively different analyses, consider how one might search for a stochastic gravitational wave signal, making use of observations in a pair of gravitational wave detectors. Over the last several years a Frequentist analysis of this problem has been developed^{1,2,3,4}. In §4.1 I outline that analysis (see also Allen’s contribution to these proceedings). In §4.2 I outline an alternative, Bayesian analysis of the same problem⁵.

4.1 A Frequentist approach

Very briefly, the Frequentist analysis developed in the literature^{1,2,3,4} begins with the observation that a stochastic signal will lead to a correlated response in the output of physically distinct detectors, while instrument noise in the same detectors is likely to be largely uncorrelated. This observation leads us

to consider Frequentist decision rules based on a correlation of the output of several detectors. Focus attention on two detectors, whose discretely sampled output over the interval $[0, T)$ are given by the sequences $h_1[k]$ and $h_2[k]$, with k running from 0 to $N_T - 1$. Focus attention on the generalized correlation

$$\mu_Q(h_1, h_2) = \sum_k^{N_T} \sum_j^{N_T} h_1[k] Q[k-j] h_2[j]. \quad (20)$$

The coefficients $Q[k-j]$ are, for now, arbitrary.

If no signal is present, then the distribution of μ_Q depends on the statistical properties of the detector noise; for instance, if the noise is uncorrelated between the detectors, then μ_Q will have vanishing mean. If the signal is present, the distribution of μ_Q depends on statistical properties of both the noise and the stochastic signal and will, in general, be different from the case of no signal because the signal leads to a correlation in the output of the two detectors. Let σ_Q^2 be the variance of μ_Q *in the absence of a signal*, i.e.,

$$\sigma_Q^2 \equiv \overline{\mu_Q^2} \quad (21)$$

where the average is over different instantiations of the random detector outputs when these are detector noise alone. Define also the *signal-to-noise ratio* ρ_Q :

$$\rho_Q(h_1, h_2) \equiv \mu_Q(h_1, h_2) / \sigma_Q. \quad (22)$$

If no signal is present and the noise in the two detectors is uncorrelated, ρ_Q will, averaged over many trials, vanish. On the other hand, if there is a signal present then ρ_Q will, over many trials, have a non-zero average value which is proportional to the signal strength and independent of the normalization of Q . A Frequentist decision rule is to fix Q and evaluate ρ_Q , deciding that a signal is present if ρ_Q exceeds a fixed threshold. The amplitude of μ_Q will, on average and for sufficiently large signal amplitudes, be proportional to the squared signal amplitude.

The freedom in the choice of Q can be used to tune the test. Suppose that the spectrum of the stochastic signal is known up to a constant, unknown amplitude. Assume that, for fixed non-zero signal strength, better tests give larger $\overline{\rho_Q}$; then, knowing the statistical properties of the signal and the noise we can maximize $\overline{\rho_Q}$ over the coefficients Q to find the “best” test of the form given in equation 20 (best in this case meaning the test that gives greatest signal-to-noise for fixed signal). Since $\overline{\rho_Q}$ is proportional to the signal amplitude, if we decide that a signal is present, ρ_Q also provides an estimate of the signal amplitude.

4.2 A Bayesian approach

Observations enter Bayesian analyses through the likelihood function (cf. eq. 15). Regard the discretely sampled detector outputs $h_1[k]$ and $h_2[k]$ as components of $\mathbf{h}[k]$, the vector-valued output of a gravitational wave receiver consisting of two detectors. Assume for simplicity that the noise and signal both have Gaussian-stationary statistics (this is the case for the usual stochastic signals considered and is also assumed in the Frequentist analyses to date^{2,3,4}). The statistical properties of the receiver noise $\mathbf{n}[k]$ are then characterized fully by the sequence of noise correlation matrices $\mathbf{C}[k]$:

$$\mathbf{C}[j-k] = \overline{\mathbf{n}[j] \otimes \mathbf{n}[k]} \quad (23)$$

$$\|\mathbf{C}[j-k]\| = \left\| \begin{array}{cc} \overline{n_1[j]n_1[k]} & \overline{n_1[j]n_2[k]} \\ \overline{n_2[j]n_1[k]} & \overline{n_2[j]n_2[k]} \end{array} \right\|. \quad (24)$$

The probability that an observation \mathbf{h} is a sample of receiver noise is a multi-variate Gaussian,

$$P(\mathbf{h}|0) = \exp(-\langle \mathbf{h}, \mathbf{h} \rangle_{\mathbf{C}}) / 2\pi \sqrt{\det \|\mathbf{C}\|} \quad (25)$$

$$\langle \mathbf{h}, \mathbf{h} \rangle_{\mathbf{C}} = \frac{1}{2} \sum_{j,k}^{N_T} \mathbf{h}[j] \cdot \|\mathbf{C}\|_{jk}^{-1} \cdot \mathbf{h}[k], \quad (26)$$

where the covariance matrix $\|\mathbf{C}\|^{-1}$ is the inverse of the $2(2N_T-1) \times 2(2N_T-1)$ block matrix $\|\mathbf{C}\|$ composed from the sequence $\mathbf{C}[k]$: $\|\mathbf{C}\|_{jk} = \|\mathbf{C}[j-k]\|$.

In the presence of a Gaussian-stationary stochastic signal the receiver output remains Gaussian-stationary; however, the covariance changes. The new covariance \mathbf{K} depends on the auto-correlation of the receiver noise \mathbf{C} and the auto-correlation of the receiver response to the stochastic signal $\mathbf{S}(\boldsymbol{\theta})$, which is characterized by $\boldsymbol{\theta}$. Thus, the probability that observed receiver output \mathbf{h} is a sample of noise plus signal is

$$P(\mathbf{h}|\boldsymbol{\theta}) = \exp(-\langle \mathbf{h}, \mathbf{h} \rangle_{\mathbf{K}(\boldsymbol{\theta})}) / 2\pi \sqrt{\det \|\mathbf{K}(\boldsymbol{\theta})\|} \quad (27)$$

$$\mathbf{K}(\boldsymbol{\theta}) = \mathbf{C} + \mathbf{S}(\boldsymbol{\theta}). \quad (28)$$

The likelihood function is the ratio of $P(\mathbf{h}|\boldsymbol{\theta}, \mathcal{I})$ to $P(\mathbf{h}|0, \mathcal{I})$, or

$$\Lambda(\mathbf{h}|\boldsymbol{\theta}) = \sqrt{\frac{\det \|\mathbf{C}\|}{\det \|\mathbf{K}(\boldsymbol{\theta})\|}} \frac{\exp(-\langle \mathbf{h}, \mathbf{h} \rangle_{\mathbf{C}})}{\exp(-\langle \mathbf{h}, \mathbf{h} \rangle_{\mathbf{K}(\boldsymbol{\theta})})} \quad (29)$$

The optimal test described in §4.1 is based on the test statistic ρ , which is ρ_Q (cf. eq. 22) with Q chosen to give the largest possible average value for ρ_Q in the presence of a signal. This statistic is, by construction, linear in the output of each of the two detectors. The likelihood function, on the other hand, has a more complex dependence on the detector outputs; in particular, $\Lambda(g|\theta)$ *is not a function of the test statistic ρ* . As a result, conclusions reached through the Frequentist test summarized in §4.1 will be quantitatively different than those reached in a Bayesian analysis.

A difference does not imply that one analysis is right and the other wrong. As we have seen, Bayesian and Frequentist analyses do not address the same questions; so, they are not required to reach “identical” conclusions. On the other hand, it may well be that one analysis is more appropriate or responsive to our concerns than the other. We can only make the choice of appropriate analysis tool when we understand the distinction between them.

5 Conclusions

Bayesian data analyses address how observations affect our *degree of belief* in propositions about nature: for example, in the proposition that radiation originating from a new supernova in the Virgo cluster was incident on a particular gravitational wave detector during a particular hour. The result of a Bayesian analysis is a quantitative measure of our degree of belief in the proposition in the face of an observation — the probability that the proposition is true.

Frequentist data analyses address our confidence in a general procedure for deciding, *e.g.*, whether a signal is present or absent. Frequentists do not assess degree of belief or confidence in any particular conclusion; rather they assess confidence in the procedure used for reaching conclusions. The result of a Frequentist analysis is a conclusion about the state of nature, made by a carefully chosen rule, upon examination of the data. *Confidence* in that conclusion is an assessment of the rule’s *average performance*: if one were to observe the same source in the same detector many times, how frequently, or by what average amount, would our procedure for deciding err?

In a Frequentist analysis, confidence is the reliability of a procedure; in a Bayesian analysis, it is the degree of belief ascribed to a hypothesis. Since astrophysical events that generate gravitational waves tend to be unique, gravitational wave data analysis that focuses on the properties of individual sources should be Bayesian. When studying individual sources we are very interested in quantifying our uncertainty (in, *e.g.*, the source chirp mass) given the only observations we have. We are generally not interested in knowing how well some procedure might do in estimating, *e.g.*, the chirp mass if we could ob-

serve the same source many times. We only get one look at most sources — stochastic background and periodic sources being the exceptions — and Bayesian techniques are more suited to the study of individual sources.

Even in the case of stochastic and periodic signals, Bayesian analyses address questions closer to our immediate interests. Given a long enough observation, Frequentist and Bayesian analyses converge to equivalent conclusions: in the Bayesian analysis, the long observation leads us to increasingly narrow the degree of our uncertainty, while in a Frequentist analysis, the long series of observations leads to a series of conclusions (from repeated application of our inference rule) whose distribution will be consistent with only one possible hypothesis about nature. In the Bayesian analysis, however, we know at any point along the way what our degree of uncertainty is, while in the Frequentist analysis our conclusions are not about our degree of uncertainty (in, *e.g.*, the presence or absence of a signal) but about how likely it is that one can distinguish between the alternatives (*e.g.*, signal absent/present, or signal amplitude) given the number of different observations made up to that point.

Acknowledgments

It is a pleasure to thank the organizers and CERN for their efforts in putting together a very successful meeting. I am also grateful for the hospitality of the LIGO Project and the California Institute of Technology, where this manuscript was prepared. This work was supported by National Science Foundation awards PHY 93-08728 and PHY 95-03084.

References

1. Peter F. Michelson. On detecting stochastic background gravitational radiation with terrestrial detectors. *MNRAS*, 227:933–941, 1987.
2. Nelson Christensen. Measuring the stochastic gravitational-radiation background with laser-interferometric antennas. *PRD*, 46(12):5250–5266, 15 December 1992.
3. Eanna E. Flanagan. Sensitivity of the Laser Interferometer Gravitational Wave Observatory to a stochastic background, and its dependence on the detector orientations. *PRD*, 48(6):2389–2407, 15 September 1993.
4. Bruce Allen and Joseph D. Romano. Detecting a stochastic background of gravitational radiation: signal processing strategies and sensitivities, September 1997. in preparation.
5. Lee Samuel Finn. Gravitational-wave data analysis with multiple detectors: The gravitational-wave receiver. II. Stochastic signals, 1997. In preparation.